VIBRATION RESPONSE OF A CYLINDRICAL SKIN TO ACOUSTIC PRESSURE
VIA THE FRANKEN METHOD  Revision E

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Introduction

The front end of a typical rocket vehicle contains avionics and a payload, enclosed by a cylindrical skin.  Rocket vehicles are subjected to intense acoustic loading during liftoff. The external acoustic pressure causes the skin surfaces to vibrate.   The skin sections are also excited by structural-borne vibration transmitted directly from the engine or motor. Nevertheless, the acoustic field is usually the dominant excitation source.

The purpose of this report to present an empirical method for calculating the vibration response of a cylindrical skin to an external acoustic pressure field.  This method is based on data collected by Franken, as documented in References 1 through 4.

The external acoustic pressure field may be due either to liftoff or to aerodynamic buffeting thereafter.

Method

A vibroacoustic response function is given in Figure 1.   The function was developed from studies of Jupiter and Titan 1 acoustic and radial skin vibration data collected during static firings.

The function predicts the skin vibration level in GRMS based on the input sound pressure level in dB, vehicle diameter in feet, and surface weight density in pounds per square foot.  The bandwidth for the predicted vibration levels is the same as the bandwidth for the acoustic input levels.  For example, the vibration levels will be in terms of one-third octave spectra if the acoustic pressure is in terms of one-third octave spectra.

Note that the function in Figure 1 predicts levels in terms of a 6 dB amplitude range.  No details are available as the statistical considerations used to arrive at this range.

Assumptions

The Franken method assumes

1.  All flight vehicles to which the procedure is applied have similar dynamic
    characteristics to the Jupiter and Titan 1 vehicles.

2.  Vibration is primarily due to the acoustic noise during liftoff or other
    pressure fields during flight which can be estimated.

3. The vibration magnitude is directly proportional to the pressure level of the excitation and inversely proportional to the surface weight density of the structure.

4. Predominant vibration frequencies are inversely proportional of the diameter of the vehicle.

5. Spatial variations in the vibration can be considered as a random variable.

VIBRATION RESPONSE OF CYLINDRICAL SKIN TO ACOUSTIC PRESSURE
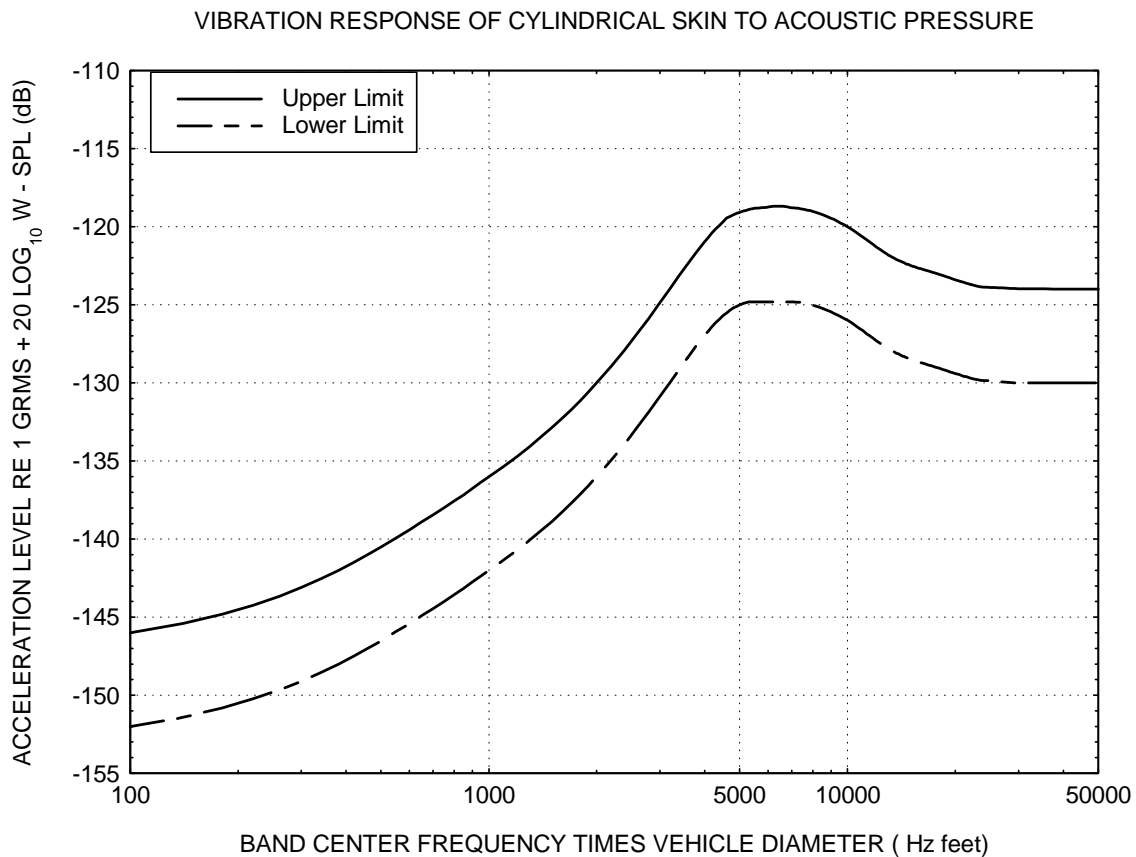


Figure 1.  Vibroacoustic Frequency Response Function

Note that W is the skin surface weight density in units of psf.  An equivalent graph in terms of metric units is given in Appendix A.

<u>Ring Frequency</u>

The ring frequency $f_r$ is the frequency at which the longitudinal wavelength in the skin material is equal to the vehicle circumference.

$$f_r = \frac{C_L}{\pi d} \tag{1}$$

where

    $C_L$    is the longitudinal wave speed or speed of sound in the skin material

    $d$    is the diameter

The longitudinal wave speed in aluminum bar is approximately 16,539 feet per second. This value increases to 20,670 feet per second for aluminum bulk. The bar value is recommended for thin-walled cylinders.

Thus the ring frequency for aluminum is

$$f_r = \frac{5264 \text{ Hz feet}}{d} \tag{2}$$

Equivalent formulas are

$$f_r = \frac{63,172 \text{ Hz inches}}{d} \tag{3}$$

$$f_r = \frac{1604 \text{ Hz meters}}{d} \tag{4}$$

Note that the curves in Figure 1 reach their respective peaks near this ring frequency.

The corresponding mode shape is one in which all of the points move radially outward together and then radially inward together. Further information on the characteristics of the ring frequency is given in Reference 5.

Speed of Sound

| Material | Density (lbm/in$^3$) | Elastic Modulus (psi) | Speed of Sound (in/sec) | Speed of Sound (ft/sec) |
|---|---|---|---|---|
| Aluminum | 0.098 | 1.0E+07 | 198,463 | 16,539 |
| Brass | 0.307 | 1.5E+07 | 137,752 | 11,479 |
| Copper | 0.322 | 1.6E+07 | 138,492 | 11,541 |
| Glass/Epoxy | 0.072 | 3.0E+06 | 126,820 | 10,568 |
| Graphite/Epoxy | 0.058 | 1.0E+07 | 257,976 | 21,498 |
| Steel | 0.283 | 2.9E+07 | 197,164 | 16,430 |
| Titanium | 0.163 | 1.6E+07 | 191,650 | 15,971 |

Table 1.  Speed of Sound

The values in Table 1 are approximate.

Note that the speed of sound values are for bar rather than bulk.

References

1.  Summary of Random Vibration Prediction Procedures, NASA CR-1302, 1969.

2.  Vibration, Shock, Acoustics; McDonnell Douglas Astronautics Company, Western Division, 1971.

3.  Himelblau, et al; Guidelines for Dynamic Environmental Criteria (Preliminary Draft), Jet Propulsion Laboratory, California Institute of Technology, 1997.

4.  R. Lyon, Random Noise and Vibration in Space Vehicles, SVM-1, The Shock and Vibration Information Center, United States Department of Defense, 1967.

5.  T. Irvine, Ring Vibration Modes, Vibrationdata Publications, 2002.
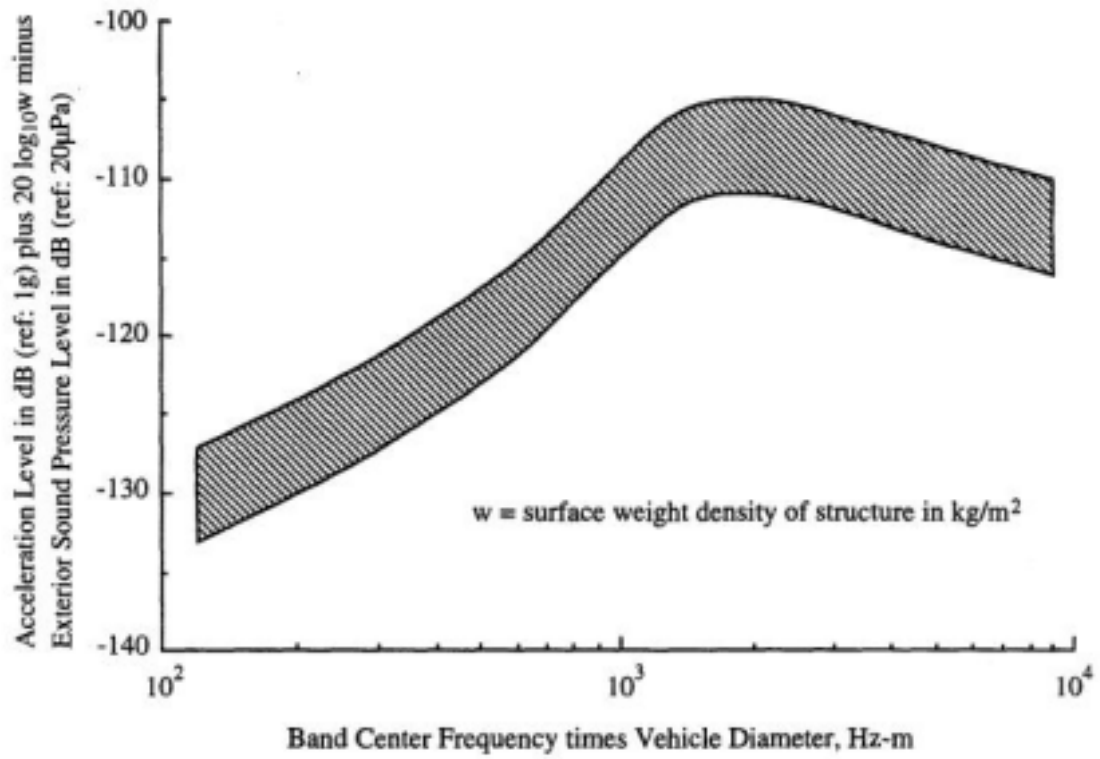
APPENDIX A



Figure A-1. Vibroacoustic Frequency Response Function, Metric Units

This graph is taken from Reference 3.

APPENDIX B


Sample Franken Method Calculation, English Units

Consider an aluminum cylinder.

The diameter is 40 inch, or 3.333 feet.

The skin thickness is 0.25 inch

The mass density of aluminum is 0.1 lbm/in^3.

The equivalent surface mass density is 0.025 lbm/in^2, or 3.6 lbm/ft^2.

Thus,

$$W = 3.6 \text{ psf} \qquad\qquad\qquad (B-1)$$


Consider that the external sound pressure level is 120 dB at a center frequency of 100 Hz, in terms of a one-third octave band.

The band center frequency at 100 Hz multiplied by the vehicle diameter in feet is

$$100 \text{ Hz x } 3.333 \text{ ft } = 333.3 \text{ Hz-ft} \qquad\qquad\qquad (B-2)$$


The Franken upper limit curve value is

$$-142 \text{ dB at } 333.3 \text{ Hz-ft} \qquad\qquad\qquad (B-3)$$


Let

F = Franken upper limit dB level

A = response acceleration level in GRMS.

The Franken formula is

$$20 \log \left( \frac{A}{1 \text{ GRMS}} \right) + 20 \log W - \text{SPL (dB)} = F \qquad \text{(B-4)}$$

$$20 \log \left( \frac{A}{1 \text{ GRMS}} \right) = F - 20 \log W + \text{SPL (dB)} \qquad \text{(B-5)}$$

Recall

| | | | |
|---|---|---|---|
| F | = | 142 dB | from Franken upper limit curve |
| W | = | 3.6 psf | from problem statement |
| SPL | = | 120 dB | from problem statement |

By substitution,

$$20 \log \left( \frac{A}{1 \text{ GRMS}} \right) = -142 - 20 \log 3.6 + 120 \qquad \text{(B-6)}$$

$$20 \log \left( \frac{A}{1 \text{ GRMS}} \right) = -142 - 11.1 + 120 \qquad \text{(B-7)}$$

$$20 \log \left( \frac{A}{1 \text{ GRMS}} \right) = -33.1 \qquad \text{(B-8)}$$

$$\log \left( \frac{A}{1 \text{ GRMS}} \right) = -1.655 \qquad \text{(B-9)}$$

$$\left( \frac{A}{1 \text{ GRMS}} \right) = 0.022 \qquad \text{(B-10)}$$

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$$A = 0.022 \text{ GRMS at 100 Hz} \tag{B-11}$$

The bandwidth at 100 Hz for the one-third octave band is 23 Hz.

The PSD level at 100 Hz is

$$\frac{\left[0.022 \text{ GRMS}\right]^2}{23 \text{ Hz}} \tag{B-12}$$

$$2.13\text{e}-05 \quad \frac{\text{GRMS}^2}{\text{Hz}} \tag{B-13}$$

By convention, this is abbreviated as

$$2.13\text{e}-05 \quad \frac{\text{G}^2}{\text{Hz}} \quad \text{(at 100 Hz)} \tag{B-14}$$

The sample calculation can be repeated for other bands, given an input sound pressure level defined over these bands.

APPENDIX C

Mass Law

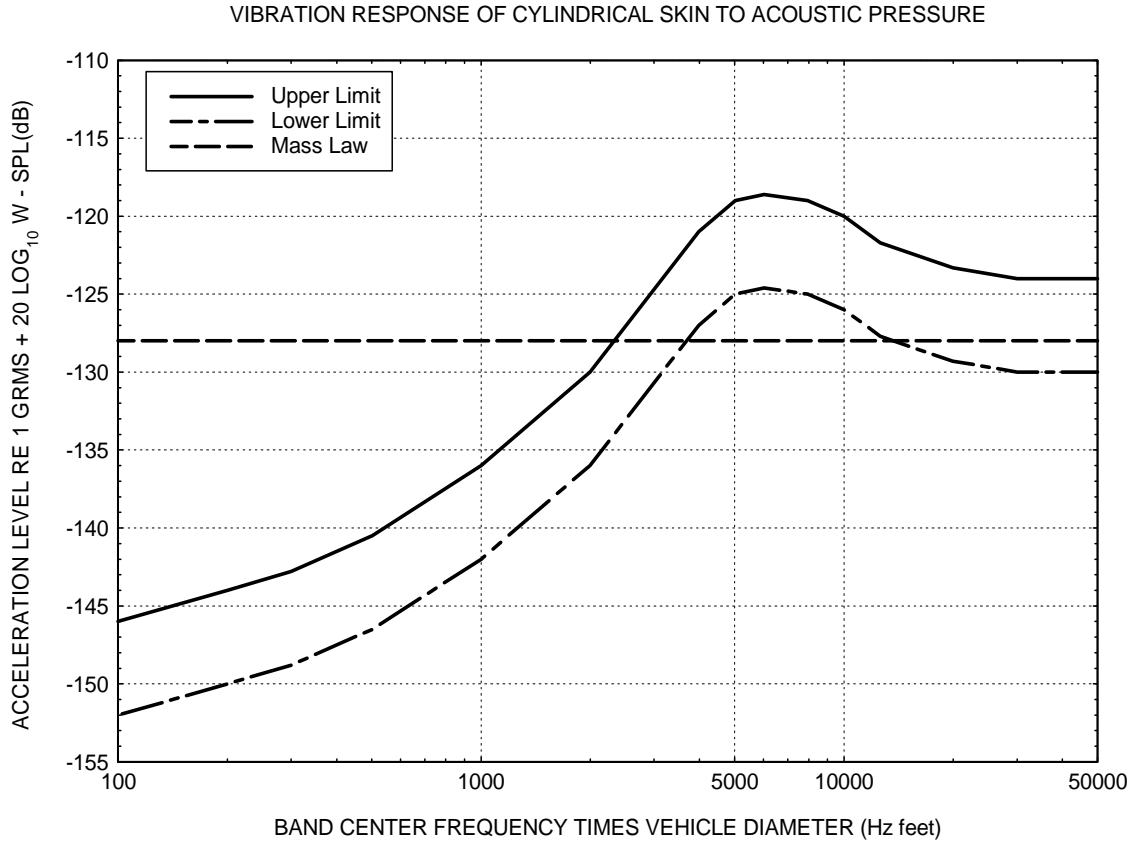VIBRATION RESPONSE OF CYLINDRICAL SKIN TO ACOUSTIC PRESSURE



Figure C-1.

The Franken curves are shown together with the mass law curve. The mass law curve assumes that the cylinder wall has mass but neither stiffness nor damping.

The mass law ML is derived as follows.

$$ML\,(dB) = 20\log\left(\frac{A}{1\ GRMS}\right) + 20\log(W) - 20\log\left(\frac{P}{Ref}\right) \qquad (C\text{-}1)$$

$$\text{ML (dB)} = 20\log\left\{\left(\frac{A}{1\ \text{GRMS}}\right)(W)\left(\frac{\text{Ref}}{P}\right)\right\} \tag{C-2}$$

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{Ref}}{1\ \text{GRMS}}\right)\left(\frac{AW}{P}\right)\right\} \tag{C-3}$$

Note that

$$\frac{AW}{P} = \left(\frac{\text{force}}{\text{mass}}\right)\left(\frac{\text{area}}{\text{force}}\right)\left(\frac{\text{mass}}{\text{area}}\right) = 1 \tag{C-4}$$

By substitution

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{Ref}}{1\ \text{GRMS}}\right)\right\} \tag{C-5}$$

The acoustic pressure reference is

$$\text{Ref} = 4.18\text{e-}07\ \text{psf} \tag{C-6}$$

$$\text{ML (dB)} = 20\log\left\{\left(\frac{\text{Ref}}{1\ \text{GRMS}}\right)\right\} \tag{C-7}$$

$$\text{ML (dB)} = -128\ \text{dB} \tag{C-8}$$